Doc Number: BCB-03-0003
Version: 0.15
Category: Calc Outline

Outline of Bare Fiber Scattering Calculation

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20 August 2003

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*Operated by the Universities Research Association under contract with the U. S. Department of Energy

Abstract

MiniBooNE has a laser system for calibration which sends light through optic fibers into the 12 m spherical detector tank. Most fibers terminate at flasks filled with gludox (sp?) which scatters the light nearly isotropically. One fiber is pointed from near the top of the tank, just through the optical barrier, and directed to near the bottom of the tank. This calculation is intended to describe the transmission and scattering of this "bare fiber" laser light. In order to keep my thoughts clear and my notation documented, I will describe the calculation here with both LaTeX and Mathematica variables described and related.

1 Introduction

The light begins at a point at (near) the inner light barrier which we will call $\vec{R}_f$ (fiber position). Light in oil travels at $v = c/n_{group}$. The optical barrier has a radius of R. The laser fires, resulting in a trigger at time t_t. We will describe the path of the emitted light by $\vec{S}(t) = \hat{s}vt$ with pathlength $s = |\vec{S}(t)|$ with t measured from t_t. The range of t is from t_e to t_{max} where

$$|\vec{R}_f + \vec{S}(t_{max} - t_e)| < R \tag{1}$$

At each point along $\vec{S}(t)$, the light will scatter with some amplitude for reaching a detector at $\vec{R}_d$. That scattered light will travel along the path $\vec{S}_s = \vec{R}_d - \vec{R}_f - \vec{S}(t_s)$ from $\vec{R}_f + \vec{S}(t_s)$ to $\vec{R}_d$ with pathlength $S_s = |\vec{R}_d - \vec{R}_f - \vec{S}(t_s)|$

For our more general calculation of the light scattering sensitivity function, we will calculate a response function describing the ratio of detected light divided by the produced light. I will begin the Bare Fiber Calculation as a direct calculation of the produced light. A photon is emitted at time t_e relative to the laser trigger time. The number of photons emitted with that laser pulse is

$$N_e(t_t) = \int A_e e^{-(\frac{t_e - t_t}{\sigma_e})^2} dt_e \tag{2}$$

The time of arrival at the phototube is

$$t_d = t_e + \frac{1}{v_{group}}(|\vec{S}(t)| + |\vec{R}_d - \vec{R}_f - \vec{S}(t)|) = t_e + t + \frac{S_s}{v_{group}} \tag{3}$$

where, of course, $t = \frac{1}{v_{group}}|\vec{S}(t)|$. The differential time dt_d is

$$dt_d = dt + \frac{1}{v_{group}}\frac{dS_s}{dt}dt = dt + \frac{1}{v_{group}}\frac{d}{dt}|\vec{R}_d - \vec{R}_f - \vec{S}(t)|dt. \tag{4}$$

$$dt = dt_d \frac{1}{1 + \frac{1}{v_{group}} \frac{d}{dt} |\vec{R_d} - \vec{R_f} - \vec{S}(t)|} = dt_d \frac{1}{1 + \frac{1}{v_{group}} \frac{dS_s}{dt}} \tag{5}$$

The scattering amplitude can, in principle, depend upon the scattering angle and the polarization of the light. We will assume that the laser light is unpolarized when it leaves the fiber. For convenience, we will use the dot product (cosine of the angle) letting the scattering amplitude be $A_s(\vec{S}(t) \cdot (\vec{R_d} - \vec{S}(t)))$. For isotropic scattering, the amplitude is independent of the angle and is given by

$$A = \frac{1}{L_s} \tag{6}$$

where L_s is the scattering length.

The solid angle, Ω, for detecting light is the detector area viewed by the scattering point divided by the distance from the scattering point squared. We will integrate the response over an angular region of the tank so the area is governed by the radius of the phototubes from the tank center and the fractional coverage of the tubes F_d (about 10% for MiniBooNE). At location $\vec{R_d}$ we have a detector area of $F_d R \cos\theta \, d\theta \, d\phi$ when viewed from the center of the tank. This is reduced by the cosine of the angle of incidence $\vec{S_s} \cdot \vec{R_d}/(S_s R)$

$$\Omega = F_d \frac{(\vec{S_s} \cdot \vec{R_d}) \cos\theta \, d\theta \, d\phi}{S_s^3} \tag{7}$$

2 Calculation Steps

Begin the calculation assuming the distribution of laser emission times in negligible so we can using a delta function for $N_e(t_t) = N_e$. The laser light at time t is

$$N(t) = N_e e^{-s/L_S} e^{-s/L_A} \sim N_e [1 - \frac{s}{L_S}][1 - \frac{s}{L_A}] \tag{8}$$

and the number of scattered photons in interval dt at time t is

$$N_s(t)dt = N_e \frac{v\,dt}{L_S} e^{-s/L_S} e^{-s/L_A} \sim N_e \frac{v\,dt}{L_S} [1 - \frac{s}{L_S}][1 - \frac{s}{L_A}] \tag{9}$$

$$N_s(t)dt = N_e \frac{v\,dt}{L_S} [1 - \frac{vt}{L_S}][1 - \frac{vt}{L_A}] \tag{10}$$

Calculating initially for an isotropic scattering, we will have light observed at $\vec{R_d}$ given by

$$N_d(t_d) = \int_0^{2\pi} \int_{\theta_1}^{\theta_2} \int_0^{t_{max}} dt N_s(t) \Omega(\vec{R_d}, \vec{S_s}(t)) \delta(t_d - t_e - t - \frac{S_s}{v_{group}}) \tag{11}$$